# Long distance coupling between two SRRs

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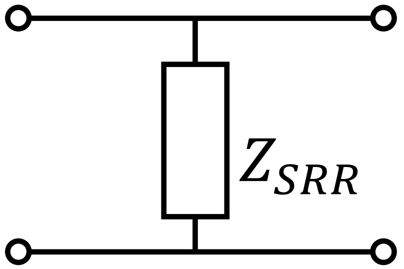
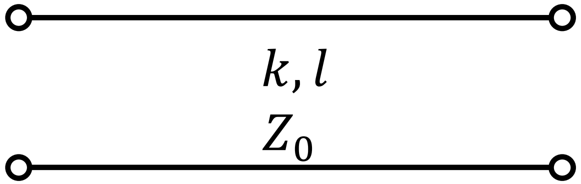
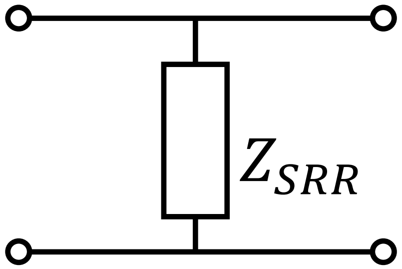
# RLC model




SRR-1




Phase Shifter




SRR-2

$$Z_{\mathrm{SSR}} = -i\frac{L}{\omega}(\omega^2 - \omega_c^2 + i2\beta\omega_c\omega)$$
$$\approx -i2L(\omega - \omega_c + i\beta\omega_c),$$

$$\Delta\Phi = \beta\Delta l$$

SRR-2 = SRR-1

$$\mathrm{M}_1 = \begin{pmatrix} 1 & 0 \\ 1/Z_{SRR} & 1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} \cos(kl) & i \cdot \mathrm{Z}_0 \sin(kl) \\ i \cdot \mathrm{Z}_0^{-1} \sin(kl) & \cos(kl) \end{pmatrix}$$

$$\mathrm{M}_3 = M_1$$

$$M = M_1 M_2 M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

# RLC model (2)

$$M = M_1 M_2 M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$S_{21} = \frac{2}{A + BZ_0^{-1} + CZ_0 + D}$$

$$S_{21} = \frac{8L^2(\omega - \omega_c + i\beta\omega_c)^2}{[8L^2(\omega - \omega_c + i\beta\omega_c)^2 + i4LZ_0(\omega - \omega_c + i\beta\omega_c)]\cos(kl) + } \cdots$$

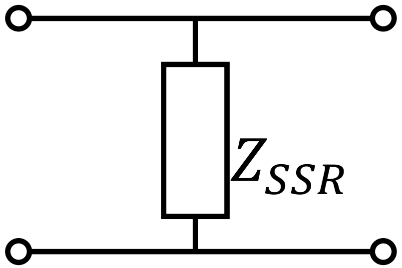
$$\cdots \frac{1}{+[i8L^2(\omega - \omega_c + i\beta\omega_c)^2 + 4LZ_0(\omega - \omega_c + i\beta\omega_c) + iZ_0^2]\sin(kl)},$$

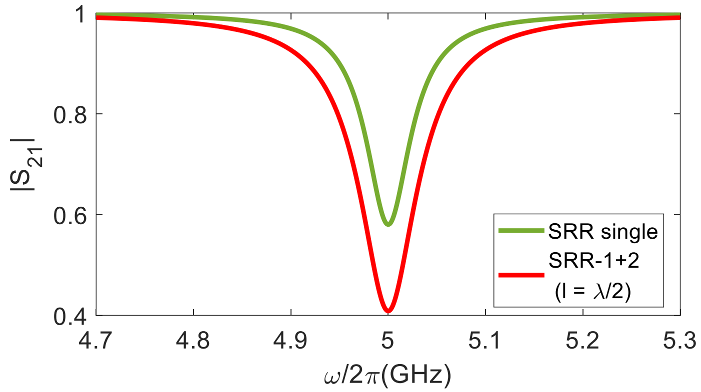
The $kl$ can be tuned by using a phase shifter.
Therefore, we have $\Phi = kl$

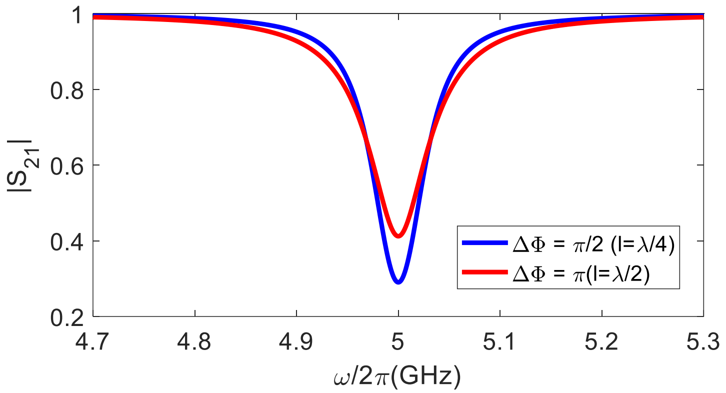
Case 1: $kl = \pi$

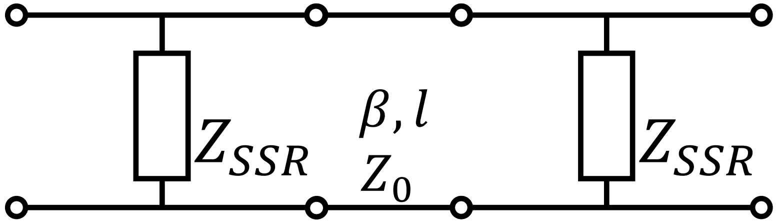
Case 2: $kl = \frac{\pi}{2}$

# Transmissions



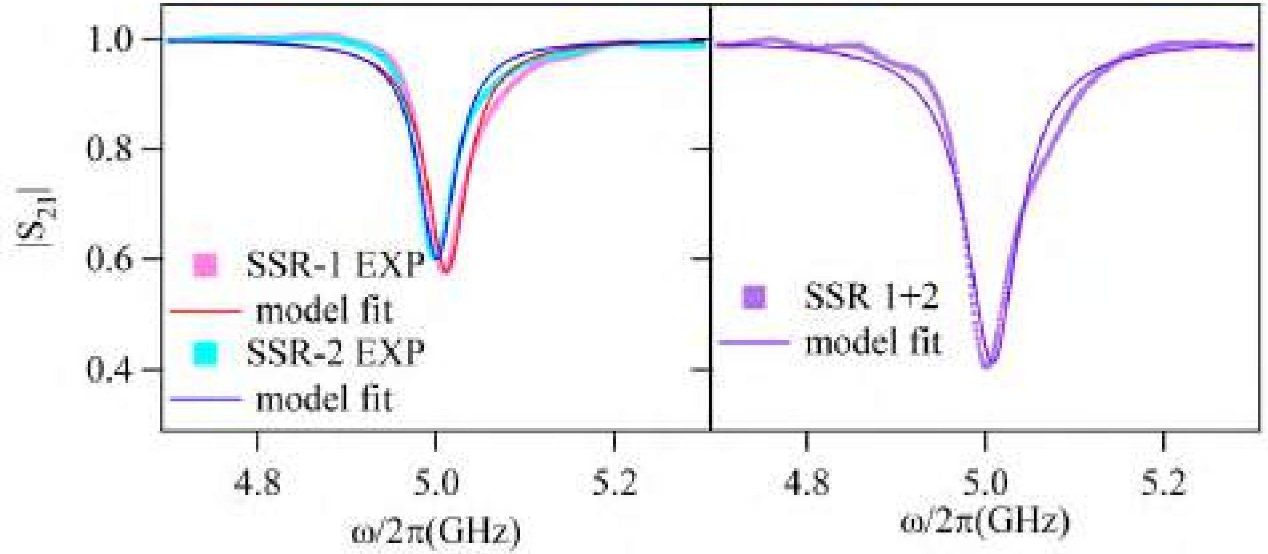

if $l = \lambda/2$

$$S_{21} = 1 - \frac{2i\Delta\omega_e}{\omega_c - \omega + i(\Delta\omega_i + 2\Delta\omega_e)}$$

if $l = \lambda/4$

$$S_{21} = i\left(1 - \frac{2i\Delta\omega_e}{(\omega - \omega_c + i\beta\omega_c + i\Delta\omega_e) - \frac{\Delta\omega_e^2}{\omega - \omega_c + i\beta\omega_c + i\Delta\omega_e}}\right)$$

# Experiments




| | $\omega_c$(GHz) | $\Delta\omega_i$ (MHz) | $\Delta\omega_e$ (MHz) |
|---|---|---|---|
| SSR-1 | 5.001 | 17.7 | 11.8 |
| SSR-2 | 5.012 | 18.4 | 13.6 |
| SSR-1+2 | 5.008 | 18.5 | 26.4 |